# Bulk properties of light deformed nuclei derived from a medium-modified meson-exchange interaction

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Deformed Hartree-Fock-Bogoliubov calculations for finite nuclei are carried out. As residual interaction, a Brueckner G-matrix derived from a meson-exchange potential is taken. Phenomenological medium modifications of the meson masses are introduced. The binding energies, radii, and deformation parameters of the Carbon,Oxygen, Neon, and Magnesium isotope chains are found to be in good agreement with the experimental data.

In recent years, one of the most exciting discoveries in nuclear physics is the neutron halo and neutron skin in light unstable nuclei near the driplines [1,2]. The new exotic nuclei provide a challenging test for theoretical models of nuclear structure. As one moves away from the magic nuclei, pairing effects become important, and the nuclear mean field may be deformed. The effective interactions employed in many models have been adjusted to reproduce the bulk properties of a limited set of stable nuclei, so that the application of such interactions to nuclei in the vicinity of the drip lines involves some extrapolations. The presently most successful models can be grouped in two classes.

(i) The Hartree-Fock-Bogoliubov(HFB) models[3,4] offer a combined treatment of the neutron Fermi energies and the pairing gaps, so that one may expect a good prediction of the neutron separation energies. HFB calculations including deformations and using a G-matrix as a residual interaction have been performed many years ago [5,6], but these calculations concentrated on binding energies, deformations, and excitation spectra, and did not emphasize the nuclear radii. In the field of exotic nuclei, HFB calculations so far have relied on efffective interactions of the Skyrme type [3].

(ii) Relativistic mean field(RMF) models[7–9] start from meson-exchange interactions which are theoretically better motivated than the Skyrme interac-

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tions. The meson coupling constants are directly adjusted to the ground state properties of nuclear matter and finite nuclei, however. The interaction NL-SH [10] includes the rho-meson exchange and can reproduce the differences between neutron and proton radii much better than previous interactions do. Deformation effects have been included, but pairing effects are usually treated in the BCS approximation [9,11].

Certainly one would like to derive the effective interaction from the bare two-nucleon interaction, but as is well-known, non-relativistic many-body theories starting from the bare two-nucleon interaction so far have not been able to reproduce the saturation properties of nuclear matter and finite nuclei. A relativistic extension of Brueckner theory can successfully predict the binding energy and the density of nuclear matter, but for finite nuclei, the problem still remains[12,13]. Recently, however, some new ideas have emerged. Brown and Rho have suggested that the masses of mesons should drop in a nuclear medium due to the broken scale invariance of QCD[14,15]. Among other effects, a considerable influence on the saturation properties of nuclear matter would result. An experimental method to search for medium modifications of meson masses is to study the production of lepton pairs in the dense and hot matter formed during heavy-ion collisions. Low-mass dilepton spectra have recently been measured by the CERES Collaborators[16]. The experiments have shown that there is an excess of dileptons with invariant masses between 250 MeV and 1 GeV. The enhancement of dileptons in this mass region may be explained by the modification of vector meson properties in the nuclear medium[17–19].

At a microscopic level, there may be many reasons for possible medium modifications of mesons. One has to recall that the so-called sigma meson employed in many One-Boson exchange potentials does not exist as a particle, but has to be interpreted as a parameterization of a correlated two-pion exchange[20,21]. Likewise, the omega-exchange of One-Boson exchange models might partly parameterize a correlated rho-pion exchange[22]. In nuclear matter, the exchange of a correlated pion pair between two nucleons is modified e.g. by the Pauli blocking of the intermediate nucleon states and by polarizations of the propagating pions due to particle-hole or $\Delta$-hole excitations. These effects would modify the properties of the effective sigma meson in the nuclear medium. Microscopic investigations of these effects so far have concentrated on the modifications of the sigma and the rho meson [23].

Given the complexity of these calculations, we feel that a more phenomenological intermediate step might be helpful in investigating the influence of medium modifications on the saturation problem. For this purpose, we suggest a model. We start from a bare two-nucleon interaction based on a meson-exchange model[24] which reproduces the nucleon-nucleon scattering data and the deuteron properties, but allow for modifications of this interaction in the

nuclear medium. We then calculate bulk properties of finite nuclei within the framework of a deformed Hartree-Fock Bogoliubov(HFB) theory. This procedure allows us to explore a wide range of nuclei where deformation and pairing effects may play an important role.

In order to solve the HFB equations in this model investigation limited to rather light nuclei, we introduce a spherical harmonic oscillator basis including the major oscillator shells up to $N = 3$ both for protons and neutrons. Since in such a small basis the results will still depend on the oscillator length b, we minimize the energy with respect to this parameter. A comparison with spherical Hartree-Fock calculations performed for $^{16}O$ in a much larger basis shows that one can expect a gain of approximately 5 MeV in the binding energy and an increase of up to 5% of the radii, but the qualitative features of the calculation persist. Time-reversal, parity, and axial symmetry are imposed on the HFB transformations. These symmetries restrict our model to the description of even-even axially deformed nuclei[25]. A full treatment of short range correlations in a finite basis is extremely involved. We therefore suggest the following approximations: a Brueckner G-matrix in momentum space is calculated for an infinite system with constant density. This G-matrix is then transformed to the finite oscillator basis in order to obtain the required two-body matrix elements of the residual interaction. Then the standard HFB equations are solved and provide the binding energies, radii, deformation and pairing parameters of the nuclei under consideration. The Fermi momentum of nuclear matter was related to the average density of the finite nucleus as follows:

$$k_F = \sqrt{\frac{3}{5}}(\frac{9\pi}{8}A)^{1/3}/\sqrt{< r^2 >}. \tag{1}$$

Here, $A$ is the nucleon number of the isotope considered, while $\sqrt{< r^2 >}$ denotes the calculated matter rms radius. Eq.(1) identifies the rms-radius of a sphere of nuclear matter with the radius $R = \sqrt{\frac{5}{3}}\sqrt{< r^2 >}$ with that of the HFB wavefunction. Selfconsistency is obtained as follows: For a given HFB wavefunction of the finite nucleus, the rms radius is calculated. Now a new Fermi momentum $k_F$ can be obtained from Eq.(1), a new G-matrix is calculated, and a new HFB wavefunction is produced. This process has to be repeated until selfconsistency is achieved. One should note that the present approximation does not correspond to a local density approximation where the force in the nuclear interior is different from that in the nuclear surface. In the present simple model, the interaction is the same over the whole nucleus. The starting energy in the G-matrix is the energy of the two initial single particle states. In the present calculation, we identify the energy of the single particle states with the average of the single particle energies obtained in the HFB calculation. Selfconsistency with respect to the starting energy

is demanded. Both direct and the exchange term of the Coulomb interaction have been taken into account.

As is well-known, in a non-relativistic framework, all the bare potentials BonnA, B, and C [24] are unable to reproduce the saturation properties of nuclear matter, let alone finite nuclei. In particular, the calculated radius of $^{16}O$ is much too small for all of these potentials. An apparently simple method to consider medium modification effects is to replace the masses of the exchanged mesons by some reduced numbers which may be fitted to the saturation properties. This procedure is only partially successful, however, since it turns out to be impossible to simultaneously reproduce both finite nuclei and nuclear matter[26]. Moreover, an unrealistically strong reduction of the meson masses is required and the results are extremely sensitive to small variations of the meson parameters.

We therefore introduce explicitly density dependent parameters, such as used by Hatsuda and Lee[27]. In a nucleus, mesons couple to nucleon particle-hole pairs. The meson masses are therefore modified by a self-energy. For sufficiently small densities and momentum transfers, the self energy is linear in the Fermi momentum $k_F$. This suggests a phenomenological modification of the meson mass proportional to $\rho^{1/3}$. Some of the mesons employed in the One-Boson exchange potential of the two-nucleon interaction partly parameterize the correlated two-pion and the correlated rho-pion exchange and therefore may have density dependencies stronger than $\rho^{1/3}$. The Brown-Rho scaling hypothesis postulates that all meson masses depend linearly on $\rho$. As a general ansatz, one might expand the meson masses in a power series with respect to $\rho^{1/3}$. Now one can define classes of phenomenological models by specific truncations of the power series. In the present investigation, we concentrate on a model which assumes a density depence of $\rho^{1/3}$ for the sigma meson mass and $\rho$ for the omega meson mass. This particular choice is motivated by the observation that the potential BonnA reproduces the binding energy of nuclear matter reasonably well, but underbinds $^{16}O$ [26]. A stronger attraction at small densities therefore seems desirable. We have checked that a medium modification of the mass of the rho-meson has quite small influences on the saturation properties. We therefore did not allow for variations of the rho-meson mass in order to keep the number of free parameters as small as possible. In this work, we choose the One-Boson exchange potential BonnA [24] as a bare two-nucleon interaction. The density dependence of the masses of the $\omega$ meson and the two $\sigma$ mesons of the potential BonnA is parameterized as in ref. [27],which makes sure that for small densities, the meson masses scale as discussed above, while for large densities, the meson masses remain positive:

$$m_\omega(\rho) = \frac{m_\omega^0}{1 + f_\omega(\rho/\rho_0)}, \tag{2}$$

$$m_\sigma(\rho) = \frac{m_\sigma^0}{1 + f_\sigma(\rho/\rho_0)^{\frac{1}{3}}}. \quad (3)$$

Here,$m_\omega^0 = 782.6$ MeV, $m_{\sigma^1}^0 = 550$ MeV, and $m_{\sigma^0}^0 = 710$ MeV are the mass parameters of the BonnA potential. The values $f_\omega = 0.036$ and $f_{\sigma^1} = 0.025$, $f_{\sigma^0} = 0.070$ for isospin 1 and 0, respectively, for the modification of the two-nucleon interaction are found by fitting the binding energy and the radius of $^{16}O$ as well as the binding energy per particle and the saturation density of nuclear matter. The density of nuclear matter $\rho_0 = 0.15 fm^{-3}$ has been introduced in order to make the parameters $f$ dimensionless. In the following, we will denote the medium-modified version of the bare two-nucleon interaction BonnA*. The sigma meson of the One-Boson exchange potential is an effective meson which parameterizes the correlated two-pion exchange. In the bare potential, different masses and coupling constants have been assigned to the $T = 0$ and the $T = 1$ channels. It is therefore not surprising that the density dependence in both channels is different. It has to be emphasized that the medium-dependence of the meson masses found in the present investigation is rather model-dependent.

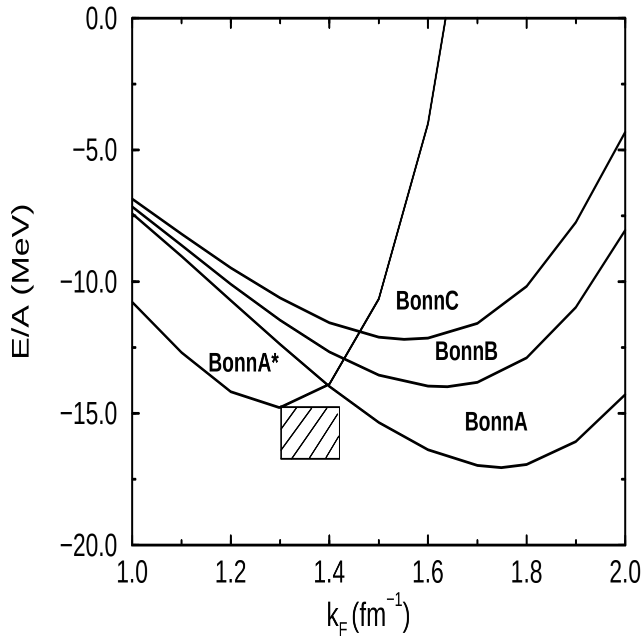


Figure 1: *Saturation properties of nuclear matter. The saturation curves are calculated in a conventional BHF method with sets of One-Boson exchange potentials[24] denoted as BonnA, BonnB, BonnC and the medium-modified meson-exchange potential BonnA*. The shaded area shows the empirical values of the saturation properties of nuclear matter.*

The nuclear matter properties are calculated with a conventional BHF approach. A discontinuous single particle spectrum, where the higher order contributions are effectively reduced, is calculated self-consistently. The calculated binding energy per particle is shown as a function of the Fermi momentum in Fig.1. The BonnA$^*$ potential leads to a binding energy per particle of $B/A = 14.6 MeV$, a Fermi momentum of $k_F = 1.29 fm^{-1}$, and a compression modulus of $K = 240$ MeV, while $B/A = 16.3$ MeV, $k_F = 1.71$ fm$^{-1}$ and $K=$ 200 MeV for BonnA.

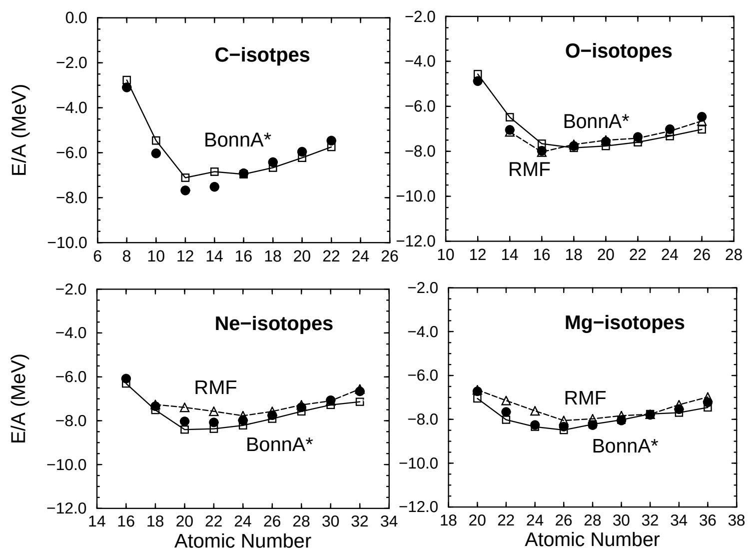


Figure 2: *Energy per nucleon of isotope chains for light nuclei : $C$, $O$, $Ne$ and $Mg$. The squares are calculated in the deformed HFB approximation with a BHF G-matrix derived from the medium-modified meson exchange potential BonnA$^*$. The triangles are results of the spherical RMF calculations with the NL-SH parameterisation. The filled solid points are the experimental data[28].*

In Fig.2, the binding energies per particle are displayed for isotopes of the $C$, $O$, $Ne$, and $Mg$ chains. The deviation from the experimental binding energies per nucleon[28] is less than 0.6 MeV for all isotopes considered. The slopes of the binding energy curves follow the experimental ones quite closely. The importance of the nuclear deformation can be seen by comparing the results of the present calculation with a spherical calculation, in particular for the isotopes $^{20}Ne$, $^{22}Ne$ and the $Mg$-isotopes. We therefore have performed spherical HFB calculations by forcing the deformation to be zero and have obtained results for the isotopes mentioned above which are very close to those obtained with the RMF-calculations. Therefore, in the figure, we only show the RMF results.

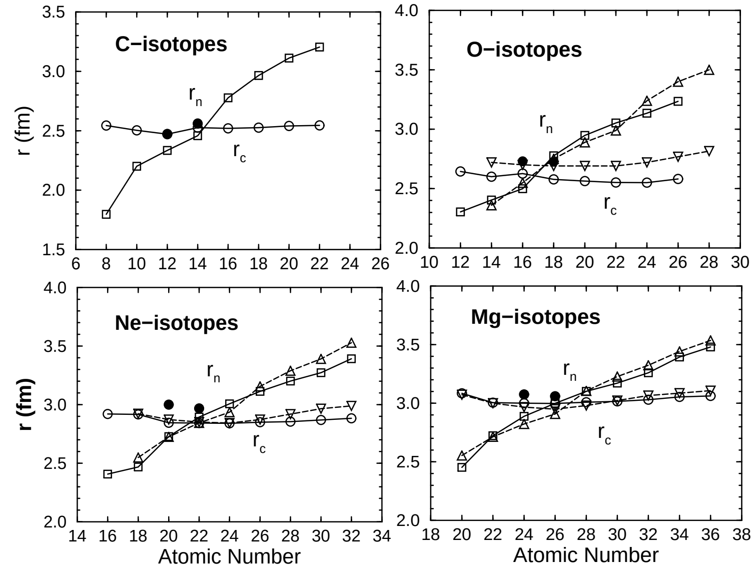


Figure 3: *Root-mean-square radii of proton charge densities and neutron point densities for the isotope chains: $C$, $O$, $Ne$ and $Mg$. The squares and circles connected with solid lines are charge and neutron rms radii, respectively, obtained in the deformed HFB with BonnA*, while the up and down triangles connected with dashed lines are those from the spherical RMF calculations with the NL-SH parameterization. The solid points are experimental data for charge radii taken from Ref.[29].*

In Fig.3, the radii of the proton and neutron density distributions are shown. The finite size of the proton has been folded in by the prescription $r_C^2 = r_p^2(calculated) + r_{nucleon}^2$ with $r_{nucleon} = 0.8fm$. There is a fair agreement with the experimental radii [29]. The present model predicts considerable neutron skins for all neutron-rich light isotopes. The slope of the RMF calculation shows a kink between the isotopes $^{22}O$ and $^{24}O$, $^{24}Ne$ and $^{26}Ne$. This can be traced to the closure of the $d_{5/2}$ shell. The HFB model predicts a smooth curve.

The calculated deformation parameters for the $Ne$- and the $Mg$-isotopes are shown in Fig.4. They are obtained from the calculated quadrupole moments by the relation

$$\beta_2 = \sqrt{\frac{4\pi}{5}} \frac{A}{Z} \frac{Q_2^{charge}}{< r^2 >_{mass}} \tag{4}$$

They are in good agreement with those from the Finite-Range Droplet Model (FRDM)[30]. The dashed lines in the figure show that two minima for the prolate and oblate shapes are obtained in the deformed HFB calculation. In the case of $^{26}Mg$, the total energy of the prolate ( $\beta_2 = 0.339$) solution is -220.71 MeV, while the total energy of the oblate ($\beta_2 = -.272$) solution is

-219.63 MeV. A relatively modest change in the effective interaction could flip the deformation of the ground state of $^{26}Mg$ from prolate to oblate.

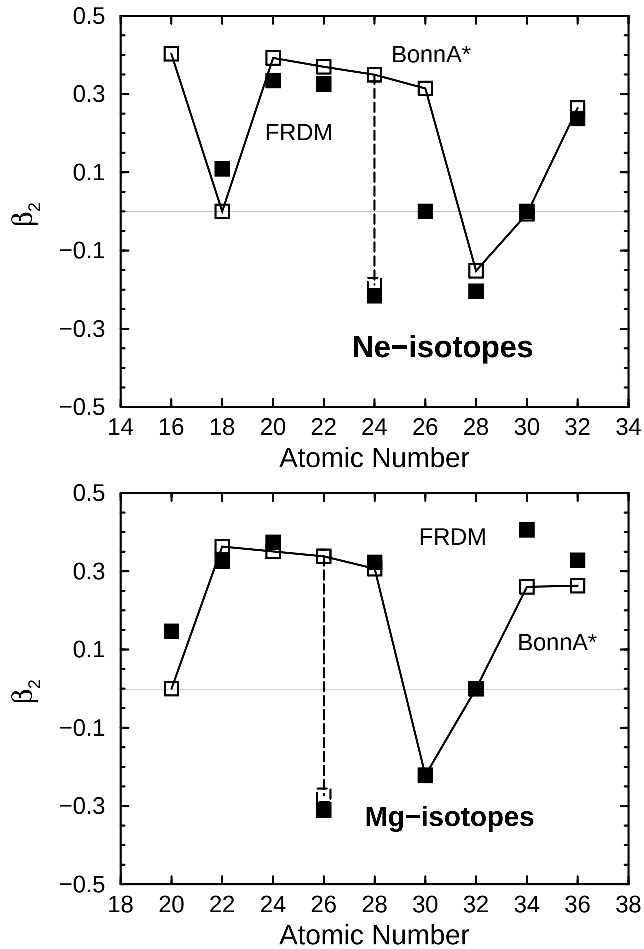


Figure 4: *Deformations for Ne- and Mg-isotopes. The notations are same as in Fig.2. The filled squares are the results of the Finite-Range Droplet Model (FRDM)[30]. The dashed lines connect two minima of the prolate and oblate deformations.*

To conclude, the results of Figs.1-4 show that a density-dependent modification of the meson masses of a bare two-nucleon interaction is sufficient to reproduce the saturation properties of both nuclear matter and finite nuclei. Despite the relatively small number of free parameters, the agreement with the experimental data is comparable to the one achieved with more phenomenological interactions. We find that the deformation parameters of the nuclei may be quite sensitive to the details of the saturation mechanism. The results of the present study are of such quality that we feel encouraged to apply this model also to isotope chains in regions of heavier masses. The density dependence of other meson parameters should be studied systematically. We believe that the present results justify the effort to develop and apply microscopic

theories of the medium modifications of all low-mass mesons.

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